MATHEMATICS – MODEL PAPER I

Time – 02 hours **No. of pages: 02** **Total Marks- 100**

Answer All

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(Q1)

(i). Prove the following by using $\epsilon - \delta$ definitions.

$$\lim_{x \to 0} x \cos\left(\frac{1}{x^2}\right) = 0$$

[5 marks]

(ii). Evaluate the following limits

(a) $\lim_{x \to 3} \frac{x^2+x-12}{x^2-9}$ (b) $\lim_{x \to 3} \frac{2x^3-54}{x-3}$ **[10 marks]**

(iii). Consider the function $f(x) = \begin{cases} \frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{x^2}, & x \neq 0 \\ 2, & x = 0 \end{cases}$

Check whether the function $f(x)$ is continuous at $x = 0$. **[10 marks]**

[**Total 25 marks**]

(Q2)

(i). Find the order and the degree of the following differential equation:

$$\frac{d^5y}{dx^5} + 4\,\frac{d^4y}{dx^4} + 3\left(\frac{dy}{dx}\right)^2 + 5y^2 = 0.$$ **[4 marks]**

(ii). Verify that $y = -3e^x + 2e^{3x}$ is a solution to the initial value problem

$$y'' - 4y' + 3y = 0, \qquad y(0) = -1, \qquad y'(0) = 3.$$ **[8 marks]**

(iii). Find the solution of the separable differential equation $(1 + x)ydx + (1 + y)xdy = 0$

[6 marks]

(iv). Solve the linear differential equation $(x + 2y^3)\frac{dy}{dx} = y$. **[7 marks]**

[**Total 25 marks**]

(Q3)

(i) At a supermarket, three customers purchase different quantities of three products. Customer A pays €34 for 2 units of Product 1, 1 unit of Product 2, and 4 units of Product 3. Customer B pays €35 for 3 units of Product 1, 2 units of Product 2, and 2 units of Product 3. Customer C pays €49 for 5 units of Product 1, 3 units of Product 2, and 2 units of Product 3. Let x, y, and z denote the prices (in euros) of one unit of Product 1, Product 2, and Product 3, respectively.

(a) Formulate a system of linear equations representing the above information. **[3 marks]**

(b) Represent the above system in the form of $A\underline{X} = \underline{B}$ where $\underline{X} = [x, y, z]$. **[2 marks]**

(c) Write down the augmented matrix. **[2 marks]**

(d) Use Gaussian elimination method to determine x, y, and z. **[12 marks]**

Continued…

(ii) Using the properties of determinant, show that

$$\begin{vmatrix} x+k & x & x \\ x & x+k & x \\ x & x & x+k \end{vmatrix} = k^2(3x+k).$$ **[6 marks]**

[Total 25 marks]

(Q4)

(i) (a) Express $\frac{1}{1-x^3}$ as partial fractions. **[4 marks]**

(b) Using the substitution $x = a\tan\theta$, evaluate $\frac{1}{a^2+x^2}dx$. **[4 marks]**

(c) Using (a), (b) find $\int \frac{dx}{1-x^3}$. **[5 marks]**

(ii). (a) Find series representation for $\frac{1}{1-x}$. **[3 marks]**

(b) Using part (a), find a power series representation for $f(x) = \ln(1+x)$. **[4 marks]**

(c) Using part(b), find the power series representation for $\int_0^x \frac{\ln(1+t)}{t}dt$. **[5 marks]**

[Total 25 marks]

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