



Mock Exam 02 (Part 01)

Subject: Introductory Statistics

Total marks: 100 marks (25 each)

Exam time: 60 min

- 1) Two sections of an introductory statistics course sit the same exam. Section X ($n = 30$) has scores summarised as: $\min = 41$, $Q1 = 58$, $\text{median} = 68$, $Q3 = 79$, $\max = 97$, $\text{mean} = 67.2$, $SD = 13.4$. Section Y ($n = 30$): $\min = 55$, $Q1 = 71$, $\text{median} = 76$, $Q3 = 82$, $\max = 89$, $\text{mean} = 75.1$, $SD = 7.6$.
 - a) For each section, determine the direction of skewness from the five-number summary. Explain your reasoning using the relative positions of the median, quartiles, and extremes.
 - b) Use the IQR rule to determine whether any scores in Section X qualify as potential outliers. Show all calculations. Explain what effect, if any, an outlier below $Q1$ would have on the mean and median of Section X.
 - c) Compare the two sections' performance using appropriate measures of centre and spread. Justify your choice of measures and state which section performed more consistently, providing a quantitative justification.

- 2) In a random sample of 400 eligible voters in a city, 228 say they intend to vote in the upcoming local election.
 - a) Verify the conditions for a one-proportion z -interval. Construct a 95% confidence interval for the true proportion of eligible voters who intend to vote. State and interpret the margin of error.
 - b) A campaign strategist wants to reduce the margin of error to ± 2 percentage points at 95% confidence. Using the sample proportion from part (a) as the planning value, determine the minimum sample size required. Show all working.
 - c) The strategist claims: "Since our interval is entirely above 50%, we can be 95% certain a majority will vote." Evaluate this claim carefully, addressing both the statistical and practical validity of the conclusion.

- 3) A physiotherapist records patients' pain scores (0–10 scale) before and after a 6-week treatment programme. For 12 patients, the mean difference (Before – After) is $\bar{d} = 2.1$ with standard deviation $SD = 1.8$. Assume differences are approximately normally distributed.



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- a) Explain why a paired t -test is more appropriate here than a two-sample t -test. State the null and alternative hypotheses using appropriate notation for the population mean difference.
- b) Compute the paired t -statistic and p -value (using $df = 11$). State your conclusion at $\alpha = 0.01$ and interpret the result in the context of the physiotherapy treatment.
- c) Construct a 99% confidence interval for the true mean reduction in pain score. Using this interval, explain whether the treatment is not only statistically significant but also clinically meaningful, assuming a 1.5-point reduction is the minimum clinically important difference.
- 4) A travel insurance policy pays €0 if no claim is made (probability 0.85), €500 for a minor claim (probability 0.10), and €3,000 for a major claim (probability 0.05). The policy is sold for €120 per customer. Claims from different customers are independent.
- a) Define the random variable $X =$ payout to a single customer. Compute $E(X)$ and standard deviation $SD(X)$. Show all steps, including the variance calculation.
- b) Define the profit per customer as $W = 120 - X$. Determine $E(W)$ and $SD(W)$ and interpret $E(W)$ in the context of the insurer's business model.
- c) For a portfolio of 500 independent policies, compute the expected total profit and its standard deviation. Using an appropriate probability model (state which and justify its use), estimate the probability that total profit exceeds €25,000.