MATHEMATICS – MODEL PAPER 2

Time – 02 hours **No. of pages: 02** **Total Marks- 100**

Answer All

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(Q1) (i) A temperature sensor indicate

$$T(x) = \begin{cases} \dfrac{|x-2|}{x-2}, & x \neq 2, \\ b, & x = 2, \end{cases}$$

where x is the actual temperature (°C).

(a). Evaluate the left-hand and right-hand limits as $x \to 2$. **[4 marks]**

(b). Does $\lim\limits_{x \to 2} T(x)$ exist? **[2 marks]**

(c). Can any value of b make the function continuous at $x = 2$? Justify your answer. **[2 marks]**

(ii) Assume that the concentration of a chemical compound in a water sample ($C(t)$) at time t is modeled by

$$C(t) = \frac{e^{3t}-1-3t}{t^2}$$

Determine the limiting concentration change as t approaches zero. **[5 marks]**

(iii) A mathematical model is used to describe a climate variation index $A(t)$ over a seasonal cycle. The index at time (t) is given by $A(t) = \cos t + (1 - \sin t)^{1/3}$, where t is measured in radians. Using the Chain Rule, determine the rate of change of the climate variation index, $\frac{dA}{dt}$. **[12 marks]**

[**Total 25 marks**]

(Q2) (i) Consider the following initial value problem:

$$(3xy + 2y^2)\, dx + \left(\frac{3}{2}x^2 + 4xy\right) dy = 0, y(1) = 1.$$

(a). Show that the differential equation is exact. **[2 marks]**

(b). Hence, find the solution satisfying the given initial condition. **[6 marks]**

(ii) A patient receives an intravenous drug at a constant rate of 10 mg per hour. At the same time, the body eliminates the drug at a rate proportional to the amount of drug present, with a proportionality constant of 0.2 per hour. Let $A(t)$ is the amount of drug (in milligrams) in the bloodstream after t hours. Assume that initially, there are 20 mg of the drug in the bloodstream.

(a). Formulate the initial value problem. **[4 marks]**

(b) Solve the initial value problem using the integrating factor method. **[10 marks]**

(c). Determine the amount of drug in the bloodstream as $t \to \infty$. **[3 marks]**

[**Total 25 marks**]

Continued…

(Q3) (i) A university book exhibition sells three types of tickets: Student ticket, Staff ticket and public ticket. Let x, y, z be the price of student, staff and public ticket respectively. During three different sessions, the numbers of tickets sold were recorded as follows:

Session	Student	Staff	Public	Total Income (Euro)
Morning Session	120	80	50	4300
Afternoon Session	90	100	60	4700
Evening Session	70	120	90	5800

(a) Formulate a system of linear equations representing the above information. **[6 marks]**

(b) Represent the above system in the form of $A\underline{X} = \underline{B}$ where $\underline{X} = [x, y, z]$. **[3 marks]**

(c) Find A^{-1} using adjoint of matrix A**.** **[12 marks]**

(d) Use part (c) to determine x, y, and z. **[4 marks]**

[**Total 25 marks**]

(Q4) (i) The concentration of a drug in blood is modeled by $C(t) = 50e^{-0.2t}$, where $C(t)$ is the concentration (mg/L) after t hours.

(a) Write down the Taylor series of e^x about $x = 0$. **[4 marks]**

(b) Use first three terms of the series obtained in par(a) to estimate the concentration after 1 hour. **[6 marks]**

(ii) The number of vehicles ($N(t)$) passing through a bridge at time t is modeled by $N(t) = 100 + 15t - 0.2t^2$, where t is measured in minutes after 7.30 a.m.

(a) Determine the total number of vehicles passing through the bridge from 7.30 a.m. to 8.00 a.m. **[10 marks]**

(b) Find the average number of vehicles passing per minute during this period. **[5 marks]**

[**Total 25 marks**]

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