MATHEMATICS – MODEL PAPER 5

Answer All **Time – 02 hours** **No. of Pages: 02** **Total Marks- 100**

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Q1) (i) A landscaper designs a curved walking path in a garden. The width of the path measured in meters at a horizontal position x is modeled by $w(x) = \sqrt{1+\sqrt{x}}$.
Determine the total area of the region covered by the walking path. **[12 marks]**

(ii) A factory releases smoke into the atmosphere. Due to the improvements in the filtration system and the gradual reduction in production activity, the rate at which smoke particles are released decreases over time. The emission rate is modeled by $R(t) = \frac{12}{(1+\sqrt{t})^3}$ kilograms per hour, where $t \geq 0$ is the time in hours after the factory begins operation. Assume that the factory continues operating indefinitely.

(a) How do you modeled the total amount of smoke released into the atmosphere as an improper integral I.
[3 marks]

(b) Using the suitable substitution evaluate the integral in Part(a). **[10 marks]**

[Total 25 marks]

Q2) (i) An engineer uses 2 × 2 matrices to represent two-dimensional measurement data associated with different operating conditions of a mechanical system. For two matrices, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ the similarity between two measurements patterns is defined by the inner product $< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$.

(a) Show that $< A\,, B >$ is an inner product on $M_{2\times 2}$, the set of all 2×2 matrices.
[10 marks]

(b) For two matrices $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$, verify the Cauchy-Schwarz inequality with respect to the given inner product. **[5 marks]**

(c) For matrices A and B, given in Part (b), verify the Triangle inequality with respect to the given inner product. **[5 marks]**

(d) Give examples of two matrices that would satisfy the Pythagorean Theorem. **[5 marks]**

[Total 25 marks]

Continued…

Q3) A veterinary clinic monitors the recovery of animals after vaccination. The temperature deviation of a vaccinated pet from its normal body temperature is modeled by a function $y(t)$, where t is the time in hours after vaccination. For a particular group of pets, including cats, dogs, and rabbits, the temperature response is assumed to satisfy the differential equation:
$2t^2y'' + 3ty' - y = 0, \; t > 0.$
It is known that one solution of this differential equation is $y_1(t) = t^{-1}$

(a) Use the method of reduction of order to find a second linearly independent solution $y_2(t)$. **[15 marks]**

(b) Obtain the general solution of the differential equation using Part(a). **[5 marks]**

(c) If the temperature deviation of a particular dog satisfies $y(1) = 2$ and $y'(1) = 5$, determine the particular solution. **[5 marks]**

[Total 25 marks]

Q4) A manufacturing company monitors three machines using three performance indicators: production efficiency, energy consumption, and vibration level. The collected data are represented by the matrix.

$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$, Here, the negative values can represent measurements below the chosen reference level.

(a) Find the singular values of matrix A. **[7 marks]**

(b) Find orthonormal eigenvectors of A. **[10 marks]**

(c) Find a singular value decomposition of A in the form $A = U\Sigma V^T$, where U is orthogonal, Σ is diagonal, and V is orthogonal. **[8 marks]**

[Total 25 marks]